

'A Solar Sine Curve'

$$a) R\cos(\theta + \alpha) = 5\cos\theta - 8\sin\theta$$

$$R\cos\theta\cos\alpha - R\sin\theta\sin\alpha = 5\cos\theta - 8\sin\theta$$

$$R\cos\alpha = 5$$

$$R\sin\alpha = 8$$

$$\tan\alpha = \frac{8}{5}$$

$$\alpha = 1.0122(4 \text{ d. p.})$$

$$R^2 = \sqrt{8^2 + 5^2} = \sqrt{89}$$

$$f(x) = \sqrt{89}\cos(\theta + 1.0122)$$

[4 marks]

$$b) \max T = 20 + \sqrt{89}$$

$$\max T = 29.4^\circ\text{C}(3 \text{ s. f.})$$

$$\text{Occurs when } \left(\frac{x}{3} + 1.0122\right) = 2\pi$$

$$x = 15.8 \text{ hours}(3 \text{ s. f.})$$

[4 marks]

$$c) 17 = 20 + \sqrt{89}\cos\left(\frac{x}{3} + 1.0122\right)$$

$$-\frac{3}{\sqrt{89}} = \cos\left(\frac{x}{3} + 1.0122\right)$$

$$\frac{x}{3} + 1.0122 = \cos^{-1} - \frac{3}{\sqrt{89}}$$

$$\frac{x}{3} + 1.0122 = 1.89, 4.39, 8.18$$

$$x = 2.65, 10.12, 21.5 \text{ hours since 10pm}(3 \text{ s. f.})$$

[4 marks]