'A Regrowing Reef'

a) $u = 4 - \sqrt{s}$, thus $\frac{du}{dh} = -\frac{1}{2\sqrt{s}}$

$$\int \frac{1}{4-\sqrt{s}} ds = \int \frac{1}{u} \times -2\sqrt{s} du$$

$$\int \frac{1}{u} \times -2\sqrt{s} du = \int \left(2 - \frac{8}{u}\right) du$$

$$\int \left(2 - \frac{8}{u}\right) du = 2u - 8\ln\ u + c$$

$$2u - 8\ln\ u + k = 2(4 - \sqrt{s}) - 8\ln\ |4 - \sqrt{s}| + c$$

Thus, $\int \frac{ds}{4-\sqrt{s}} = -8\ln\ |4 - \sqrt{s}| - 2\sqrt{s} + 8 + c = -8\ln\ |4 - \sqrt{s}| - 2\sqrt{s} + k$

[6 marks]

b)

$$\frac{ds}{dt} = 0 = t^{0.25}(4 - \sqrt{s})$$

$$4 - \sqrt{s} = 0, s = 16$$

The reef cannot have a negative surface area, therefore:

$$0 \leq s \leq 16$$

[2 marks]

c) $\frac{ds}{dt} = \frac{t^{0.25}(4-\sqrt{h})}{20}$

$$\int \frac{20}{4-\sqrt{s}} ds = \int t^{0.25} dt$$

$$20(8 - 2\sqrt{s} - 8\ln\ |4 - \sqrt{s}|) = \frac{4}{5}t^{1.25} + c$$

$$c = 20(8 - 2 - 8\ln\ 3)$$

when $s = 12$: $\frac{4}{5}t^{1.25} = [20(8 - 2\sqrt{12} - 8\ln\ (4 - \sqrt{12})) - 20(6 - 8\ln\ 3)]$

$$t^{1.25} = \frac{5}{4} \times [20(8 - 2\sqrt{12} - 8\ln\ (4 - \sqrt{12})) - 20(6 - 8\ln\ 3)]$$

$$t^{1.25} = 221(.2795202)$$

$$t = \sqrt[1.25]{221} = 75 \text{ years } (2s.f.)$$

[7 marks]